GAS CYCLES ARTICLE

In the video 'Cycles that work' I described how we could take a snap-shot of an engine or a pump to allow us to perform some analysis of them. The resultant assumption of a quasi-static equilibrium process will lead to an over-estimate of work output for engines and an under-estimate work input for compressors and pumps.

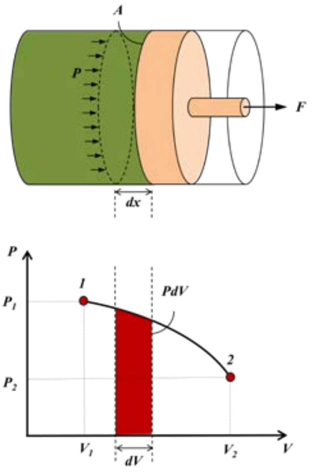

Figure 1: Expansion of gas in a cylinder pushing out a piston (top) and the corresponding gas states in the pressure-volume domain (bottom).

So, let's try some simple analysis. We can idealise the work done in the power stroke of an engine by considering gas expanding in an insulated cylinder and pushing out the piston. I have shown this schematically in Figure 1 with a pressure-volume plot below showing the change in state of the gas from 1 to 2.

We can apply the first law of thermodynamics to the gas inside the cylinder

$$(Q_{in} - Q_{out}) + (W_{in} - W_{out}) + \left(E_{mass,in} - E_{mass,out}\right)$$
$$= \Delta U + \Delta KE + \Delta PE \qquad [1]$$

The cylinder is insulated so there will be no heat transfer, i.e. $Q_{in} = Q_{out}$, and no mass transfer can occur so $E_{mass,\,in} = E_{mass,\,out}$. If the system is stationary and does not change position then we can assume that $\Delta KE = \Delta PE = 0$, so that

$$(W_{in} - W_{out}) = \Delta U \qquad [2]$$

And if we consider the piston, the work done on the piston is the force, F multiplied by the distance moved, which for an infinitesimal distance, dx is

$$dW = Fdx \qquad [3]$$

where 'd' means small change in...' and the force is the pressure in the cylinder, P multiplied by the cross-section area of the piston, A; hence

$$dW = Fdx = PAdx \qquad [4]$$

Now, Adx is the change in volume of the gas, i.e.

$$dW = Fdx = PAdx = PdV \qquad [5]$$

This quantity, PdV is the area under the curve of the gas pressure as a function of its volume, i.e. the red area in figure 1. In order to obtain the total work done by the gas we must integrate over the change in volume of the cylinder to give

$$W = \int_1^2 PdV \qquad [6]$$

where the symbol $\int$ means to integrate or sum all the small products, $P \times dV$. We can only perform this integration if we know the path taken by the gas from 1 to 2 in the pressure-volume domain.

Gases can take more than one path in the pressure-volume and some examples are shown in figure 2 together with some values of the work that might be done as the gas moves along the different paths from state 1 to state 2.

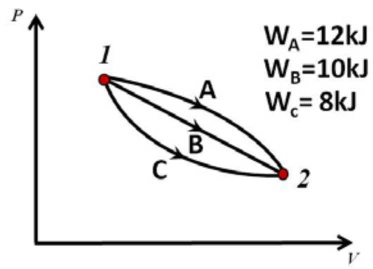

Figure 2: Choice of paths through pressure-volume domain from 1 to 2.

We can arrange to expand a gas from 1 to 2 along path A in figure 2 and gain 12kJ of work output and then compress the gas back from 2 to 1 along path C and only do 8kJ of work on it. This will give us a net gain of 4kJ of work making this a useful gas cycle, as shown in figure 3. In ideal gases, the pressure and volume are usually related by a function of the form

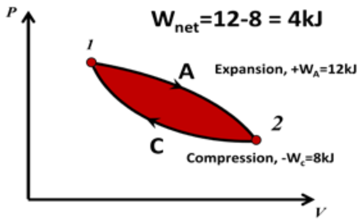

Figure 3: Simple gas cycle based on paths shown in figure 2

$$PV^{\gamma} = constant \qquad [7]$$

where γ is the ratio of the specific heat capacities, i.e. $\gamma = {}^{c_p}/_{c_v}$ and for air we can assume γ= 1.4 in standard ambient conditions.